WORK, POWER, and ENERGY

Work
- **Work** $\Rightarrow$ a force acting over a distance.
  - Work is only done by forces acting parallel to the displacement.
  - Ex: a vertical force does no work on something displaced horizontally.

\[ W = Fd\cos\theta \]

<table>
<thead>
<tr>
<th>Variable</th>
<th>Meaning</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>Work</td>
<td>Joules (J)</td>
</tr>
<tr>
<td>F</td>
<td>Force</td>
<td>Newtons (N)</td>
</tr>
<tr>
<td>d</td>
<td>Displacement/distance</td>
<td>Meters (m)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle between the force and displacement vectors</td>
<td>Degrees (°)</td>
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- 1 Joule $= 1$ N·m
- **To do WORK, a force must cause a displacement.**
  - If the force you are looking at did not cause the displacement, that force did no work.

- When a force is acting on an object to slow it down (accelerate it in the negative dir’n), **negative work** is done.
  - Ex: when you apply the brakes in your car, the brakes are doing negative work on your car.
  - Mathematically it makes sense, too: \( \cos (180^\circ) = -1 \)
Conceptual Example 1

Is work done in the following scenarios?

- Joe applies a force to the wall and becomes exhausted.
- A book falls from the table and free falls to the ground.
- A waiter carries a tray full of meals above his head by one arm straight across the room at a constant speed.
- A rocket accelerates through space.

Conceptual Example 2

The Moon revolves around the Earth in a circular orbit, kept there by gravitational force exerted by the Earth. Does gravity do positive, negative or no work on the Moon?

Example Problem 1

Workers load a 500 kg safe onto an elevator that lifts it 90 m to the 20th floor of an office building. How much work did the elevator do on the safe?

Power

- Work only takes into account forces acting on objects over a distance.
  - Notice the lack of concern for how quickly it happens.

- **Power** \( \rightarrow \) rate at which work is done.
  - In the example above, Dan has a better power rating than Bob.
\[ P = \frac{W}{t} \]

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<tr>
<td>P</td>
<td>Power</td>
<td>Watts (W)</td>
</tr>
<tr>
<td>W</td>
<td>Work</td>
<td>Joules (J)</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>Second (s)</td>
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</table>

\[ P = Fv \]

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</tr>
<tr>
<td>F</td>
<td>Force</td>
<td>Newtons (N)</td>
</tr>
<tr>
<td>v</td>
<td>Velocity</td>
<td>Meters per second (m/s)</td>
</tr>
</tbody>
</table>

- 1 Watt = 1 J/s = 1 kg\cdot m^2/s^3
- Another common measurement of power is **horsepower (hp)**.
  - 1 hp = 746 W
  - Originally adopted by James Watt in the 18th C. to compare the power of steam engines to draft horses.
- Machines, people, animals, etc. all have **power ratings**.
  - The faster you can do work, the better your power rating.

**Example Problem 2**

A 70 kg jogger runs up a long flight of stairs in 4.0 s. The vertical height of the stairs is 4.5 m. Estimate the jogger’s power output in both watts and horsepower.

**Energy**

- **Energy** → the capacity to do work.
  - If there is no energy present, work cannot be done.
- Measured in Joules (J).
- **Potential Energy** → The energy stored due to an object’s position.
  - **Gravitational potential energy** → energy stored as a result of an object’s vertical position.
  - **Chemical potential energy** → energy stored in chemicals. Ex: a barrel of oil, the human body, etc.
  - **Spring potential energy** → energy stored by compressing a spring.
  - **Elastic potential energy** → energy stored in an elastic material as it is stretched.

**Gravitational Potential Energy (GPE)**

- As an object rises in height, or vertical position, it gains GPE as a result of the Earth’s gravitational attraction to the object.
- GPE is dependent on two variables: mass and height.
  - As height increases, GPE increases
  - As mass increases, GPE increases

\[ GPE = mgh \]

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<tr>
<td>GPE</td>
<td>Gravitational potential energy</td>
<td>Joules (J)</td>
</tr>
<tr>
<td>m</td>
<td>Mass</td>
<td>kilograms (kg)</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration due to gravity</td>
<td>Meters per second squared (m/s²)</td>
</tr>
<tr>
<td>h</td>
<td>Height/vertical position</td>
<td>Meters (m)</td>
</tr>
</tbody>
</table>

- Height is typically measured from the ground, but keep in mind that it’s completely arbitrary.
  - In lab, we may set zero height at table level.
  - GPE is more concerned about the difference in heights than a firm position value from some universal h = 0 m. (Because h = 0 is made up by us, it’s arbitrary).

**Example Problem 3**

A cart is loaded with a brick and pulled at constant velocity along an inclined plane to the height of 0.45 m. If the mass of the loaded cart is 3.0 kg, then what is the potential energy of the loaded cart at the height of the trip?

**Kinetic Energy (KE)**

- **Kinetic Energy** \( \rightarrow \) the energy of motion.
  - Every moving object has kinetic energy.
  - Depends on the mass and velocity of the object.

\[ KE = \frac{1}{2}mv^2 \]

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<td>KE</td>
<td>Kinetic energy</td>
<td>Joules (J)</td>
</tr>
<tr>
<td>m</td>
<td>Mass</td>
<td>kilograms (kg)</td>
</tr>
<tr>
<td>v</td>
<td>Velocity</td>
<td>Meters per second (m/s)</td>
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- A scalar quantity (i.e., direction doesn’t matter).

**Example Problem 4**

A gun fires a bullet of mass 25 g at a muzzle velocity of 320 m/s. What is the kinetic energy of the bullet?
**Total Mechanical Energy (TME)**

- **TME** → the total amount of kinetic and potential energies present in a system.
- An object that has mechanical energy is able to do work.
  - Enables an object to apply a force to another object.

\[
TME = KE + GPE
\]

**Law of Conservation of Energy**

- The Law of Conservation of Energy → **Energy can neither be created nor destroyed.**
  - Without outside interference, energy can’t enter/leave a system.

\[
TME_i = TME_f
\]

- Energy can *change forms* within the system, but the total amount of energy before must be the same as the total amount of energy after.
Example Problem 5

Two cars of weight 70.2 kN and 15.54 kN are travelling along horizontally at a rate of 97 km/h when they both run out of gas. Luckily, there’s a town in a nearby valley, but it’s just beyond a 34.2 m high hill. Assuming that friction can be neglected, which of the cars will make it to the town?

Work-Energy Theorem

- If outside forces do work on an object, then the TME can change.
- Work-Energy Theorem → work done by an outside force is equal to the change in the kinetic energy of a system.

\[ W = \Delta KE = KE_f - KE_i \]

Example Problem 6

A 1000-kg car traveling with a speed of 25 m/s skids to a stop. The car experiences an 8000 N force of friction. Determine the stopping distance of the car.
Example Problem 7

A shopping cart full of groceries is sitting at the top of a 2.0-m hill. The cart begins to roll until it hits a stump at the bottom of the hill. Upon impact, a 0.25-kg can of peaches flies horizontally out of the shopping cart and hits a parked car with an average force of 500 N. How deep a dent is made in the car (i.e., over what distance does the 500 N force act upon the can of peaches before bringing it to a stop)?